2020

ECONOMICS — HONOURS

Seventh Paper

(Group - A)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Section - A

(Marks : 20)

1. Answer ***any two*** questions :

(a) The joint probability density function of the random variables X and Y is given by :

$$f(x, y) = \begin{cases} \frac{1}{3}(x+y), \ 0 \le x \le 1, \ 0 \le y \le 2 \\ 0 \text{ otherwise} \end{cases}$$

Find the conditional density function of X when $Y = \frac{1}{2}$. Hence find $E\left(X / Y = \frac{1}{2}\right)$. 5+5

(b) If $X_1, X_2, ..., X_n$ is a random sample from a normal population $N(\mu, \sigma^2)$, how are the following distributed? 10

(i) $\frac{X_1 + X_2 + ... + X_n}{n}$

(ii) $\frac{\overline{X} - \mu}{\sigma / \sqrt{n}}$

(iii) $\frac{\overline{X} - \mu}{s' / \sqrt{n}}$ where $s'^2 = \frac{\sum\left(X_i - \overline{X}\right)^2}{n - 1}$

(iv) $\sum\left(X_i - \overline{X}\right)^2 / \sigma^2$

(c) If $\overline{x}$ denotes the sample mean, what is the probability that $3(\overline{x} - \mu) \ge 4$ if a random sample of size 15 is taken from a $N(\mu, \sigma^2)$ with $\sigma^2 = 4$? 10

(d) Argue whether the following statement is true / false :

In the context of a 2-variable regression model $r^2 = \frac{[Cov(x,y)]^2}{Var(x).Var(y)}$ is equivalent to the expression $r^2 = \frac{ESS}{TSS}$. 10

(e) What is the distinction between an error term and a residual? 10

(f) Explain the effect of autocorrelated error on the ordinary least square estimators in a classical linear regression model. 10

(g) What is moving average? When is it ideal in determining trend in a time series? 10

(h) Let e_i be the residual in the least squares fit of Y_i against X_i (i = 1, 2, ..., n).

Derive the following results :

(i) $\sum_{i=1}^{n} e_i X_i = 0$ (ii) $\sum_{i=1}^{n} \hat{Y}_i e_i = 0$ 5+5

Section - B

(Marks : 30)

Answer ***any three*** questions.

2. Find the maximum likelihood estimators of the Mean and Variance of a normal population when both are unknown. Are they unbiased? 6+4

3. X_1, X_2, X_3 is a random sample (with replacement) of size 3 from a population with mean value μ and variance σ^2. T_1, T_2 and T_3 are the estimators of μ, where $T_1 = X_1 + X_2 - X_3$, $T_2 = 2X_1 + 3X_3 - 4X_2$, and $T_3 = \frac{\lambda X_1 + X_2 + X_3}{3}$.

(a) Check the unbiasedness of T_1 and T_2.

(b) Find the value of λ such that T_3 is an unbiased estimator of μ.

(c) Which is the best estimator? 3½+3½+3

4. A random sample of 9 experimental animals, under a certain diet give the following results :

$\sum_{i=1}^{n} x_i = 45$, $\sum_{i=1}^{n} x_i^{\,2} = 279$ where x_i denotes the weight of the i-th animal in kg. Assuming that the weight is normally distributed as a $N(\mu, \sigma^2)$, test the hypothesis $H_0 : \mu = 6$ against $H_1 : \mu < 6$.

[Given : $P\{t_8 \geqslant 1{\cdot}86\} = 0{\cdot}05$ from student's t table] 10

5. (a) Determine whether the following models are linear in parameters, or in variables, or both. Which of these models are linear regression models?

(i) $Y_i = \alpha + \beta\left(\frac{1}{X_i}\right) + u_i$

(ii) $Y_i = \alpha + \beta \ln X_i + u_i$

(iii) $\ln Y_i = \alpha + \beta X_i + u_i$

(iv) $\ln Y_i = \ln \alpha + \beta \ln X_i + u_i$

(b) Based on monthly data, the following regression results were obtained :

$$\hat{Y}_i = 0{\cdot}00681 + 0{\cdot}75815 X_i$$

$$s.e. = (0{\cdot}02596) \quad (0{\cdot}27009)$$

$$r^2 = 0{\cdot}4406$$

Find out the number of observations (n). 6+4

6. Write down the assumptions essential for each of the following tasks :

(a) For proving that OLS estimates of the parameters are unbiased.

(b) For proving that the OLS estimates are efficient.

(c) For carrying out t test. 3½+3½+3

7. On the basis of annual production figures (in thousand tons) of an industry for the years 2000-2006, the following linear trend fitted to the annual data is obtained :

$$Y_t = 107{\cdot}2 + 2{\cdot}93t,$$

where t = year with origin at 2003

Y_t = annual production in time period t.

(a) Use this equation to estimate the annual production for the years 2001 and 2007.

(b) The quarterly variations are given as :

Quarter	Seasonal Index
Jan – March	125
April – June	105
July – September	87
October – December	83

Use the indices to estimate the production during the first quarter of 2007. 4+6

8. Ages of 5 persons have been recorded as (in years) 14, 19, 17, 20, 25. For random samples of size 3 drawn without replacement from this population, obtain the sampling distribution of sample mean $(\bar{x})$. Show that the mean of $\bar{x}$ equals the population mean and obtain the standard error of $\bar{x}$. 3½+3½+3

9. (a) Given the following data :

$$\Sigma x_i y_i = 200,\ \Sigma x_i^2 = 100,\ \Sigma y_i^2 = 500,\ \overline{X} = 100,\ \overline{Y} = 150,\ n = 27$$

Estimate the parameters in the model :

$$Y_i = \alpha + \beta X_i + U_i$$

(b) Show that TSS = ESS + RSS. 6+4